

LATE-WAKE FLOWS IN STRATIFIED FLUIDS

Background. It is a common fact that any object moving in a fluid leaves a wake behind itself. If the fluid is homogeneous then the flow pattern in the wake is determined by body Reynolds number, $Re = UD/\nu$, where U and D are the velocity and size of the body, and ν is the kinematic viscosity of the fluid. With the Reynolds number increasing, the flow becomes more and more irregular, developing eventually into a highly turbulent wake¹. This remains true for towed (momentum wake) as well as for self-propelled (zero-net-momentum wake) bodies. It was suggested that in the far field (at large distances from the source) the flow is self-similar², i.e. the mean velocity and turbulence intensity profiles evolve the same self-preserving shapes in the wake, even though the particular form of the profiles is determined by the integral characteristics of the flow (e.g., net linear and/or angular momentum).

When the fluid is stratified, the wake formation and development is significantly influenced by density gradient. Additional parameters, body Froude number, $Fr = U/ND$, and non-dimensional time, Nt , also become important (here, N is the buoyancy frequency and t is time). In many applications, Re and Fr can be very large near the source of motion, but at late stages of the flow development with typical fluid velocities and length scales U^* and D^* , the global Re^* and Fr^* numbers become small³. In that sense, stratification serves to stabilize the late flow. Experimental studies of momentum stratified wakes behind three-dimensional bodies demonstrate that the far wake consists of long lasting patches of vertical vorticity of opposite signs ("pancake" eddies) organized into a vortex street, the horizontal length scale and velocities of which are much larger than those in the vertical direction^{4,5} (quasi-two-dimensional flow). In addition, the structure of the late-wake flow does not significantly depend on the shape of the towed body when Re exceeds a critical value⁶.

The structure of zero-net-momentum stratified turbulent wakes was studied much less^{7,8}. For a self-propelled body moving with a constant velocity the drag is balanced by thrust, so that the total momentum imparted to the fluid is zero. The organization of far-wake eddies is qualitatively different from that in the case of a towed body. Also, they are characterized by smaller horizontal scale and spacing. Reliable data have been obtained only for the wake of a steadily moving ($U = \text{const}$) self-propelled body. In practice, self-propelled underwater vehicles frequently accelerate/decelerate or change the direction of motion, thus transporting significant horizontal momentum to the surrounding fluid. As a result of such kind of maneuvers, the generation of unusually large eddies in the late wake occurs during periods of unsteady motion⁶. This owes to the fact that the horizontal momentum, imparted locally to the stratified fluid, has a strong tendency to generate large vortex structures, which have much larger horizontal length scales and significantly different characteristics compared to those produced in the late wake during steady motion.

The other important aspect of the late-wake stratified flows is the organization of large eddies. Depending on the forcing, large eddies differ in their morphology, i.e. the number of elementary constituting vortices (monopoles, dipoles, etc.). Momentum source (force) generates a vortex dipole, whereas a force doublet with zero net momentum generates a quadrupolar flow³. Morphology determines the evolution of large eddies. Dipolar eddies, which possess a linear momentum, can propagate quite far from their origin, while monopolar eddies have only angular momentum and, therefore, simply follow the mean current. When linear and angular momentum are not equal to zero, the evolution of the resulting eddy is strongly influenced by their ratio, and a whole family of isolated (zero net vorticity) self-propagating eddies may be generated⁹.

The nature of the late wake produced by underwater vehicles can have direct effect on their performance and maneuverability.

Statement of the Problem. The analysis of experimental data suggests, that linear and/or angular momentum are transported to the fluid during a maneuver (unsteady motion) of the underwater vehicle, which leads to the formation of unusually large eddies in the late wake. Estimates show, that when oceanic underwater vehicle changes its velocity by 10% or its direction by 5 degrees, large eddies of characteristic size 1 - 2 km and with the decay time of about a day may be expected. It might be conjectured, that the integral characteristics of the flow, such as net linear and angular momentum (and higher moments of force distribution) which are conserved during flow evolution, actually govern its development at late times. The details of initial conditions are of minor importance. Because the integral characteristics of the flow are predetermined by the source conditions, the general problem of the relation between the forcing (localized spatial-temporal distribution of forces) and the morphology of the vortices arises.

At the same time, the evolution of late-wake large eddies is influenced by environmental conditions (e.g., shear, non-uniform stratification, background turbulence) which may even prevent the formation of large eddies¹⁰. Therefore, the second problem, how do external factors affect the formation and evolution of late-wake vortex structures, becomes a logical course of the first one.

Besides obvious scientific interest, such effects may have important potential applications and have not been studied previously.

Goal. Develop through laboratory experiments and theoretical analysis a basic understanding and a predictive model of the generation, evolution and decay of large eddies formed in the late wakes of unsteadily moving underwater vehicles.

Approach to the Problem. In order to study the dependence of large eddy morphology on the source conditions, it is proposed to model forcing by a combination of 'point' momentum sources (jets) moving with arbitrary velocity in the fluid. To reproduce rather complex action of an unsteadily moving self-propelled body on a fluid, a novel approach of using a moving force doublet is proposed (Fig. 1).

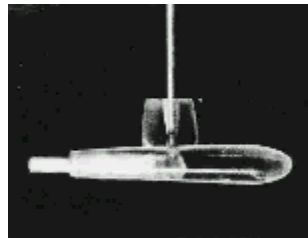


FIG. 1. Model of a submarine (body + jet). Length 3.5 cm, effective diameter 0.7 cm, nozzle diameter 0.15 cm.

This approach helps to address such aspect of the problem, which is currently not well described and understood, as how large eddies are actually generated under unsteady and maneuvering conditions.

Experiments are conducted in: (a) tow tank which is equipped with a computer-controlled driving servomechanism, to translate the test platform at a specified time-dependent velocity, (b) 'Odell-Kovaszny' water channel, where arbitrary velocity profile (vertical shear) may be realized. Both tanks are filled with a density-stratified fluid (salt water). Linear, two-layer and more complicated density distributions are used to mimic different stratification profiles in the upper ocean. Using standard techniques (video, shadowgraph, thymol-blue dye visualization, conductivity micro-probes, etc.), laboratory measurements of parameters such as flow patterns in the near and far field, their size, propagation velocity and distributions of density are obtained. Using more sophisticated techniques, such as particle image velocimetry (PIV), the full-field velocity and vorticity distributions in the flow may be obtained. This will help to clarify the dynamics and mechanisms of eddy formation.

Conducted experiments (Fig. 2, 3) clearly demonstrate the difference in the morphology of the self-propelled (steady) and momentum (accelerated) late-wake flows.

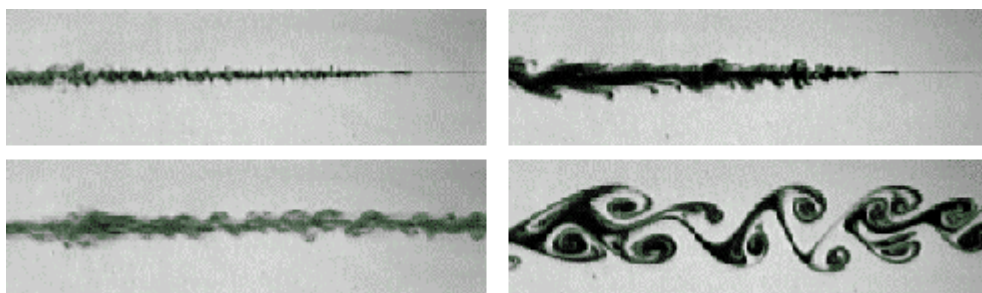


FIG. 2. Balanced (left) and accelerated (right) self-propelled wakes in a stratified fluid. Model (Fig. 1) is used to simulate self-propelled wakes. Near (top) and late (bottom) wake patterns are shown.

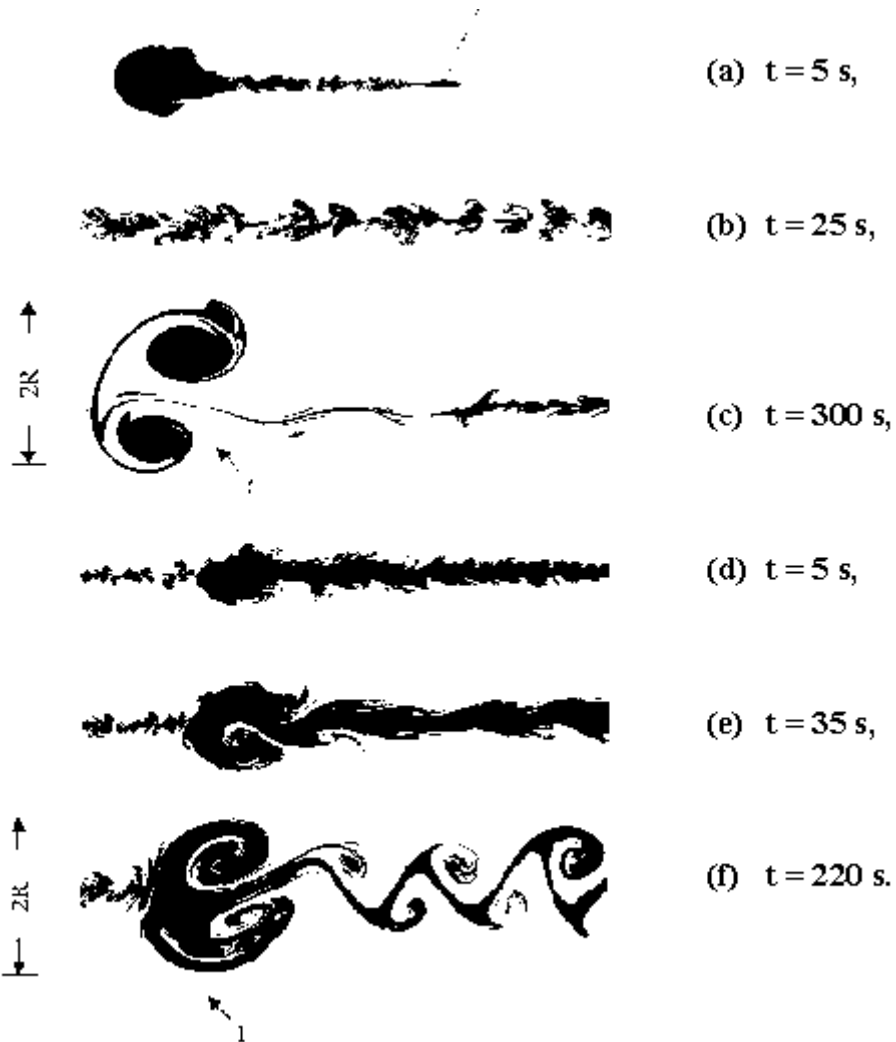


FIG. 3. Top view flow patterns in a stratified fluid ($N=1 \text{ rad s}^{-1}$) when the model: (a) starts moving from left to right, (b) moves with constant speed, (d) accelerates again. With time large eddy (1) is formed in the late starting flow (c). Similar structure (1) is formed in the late momentum wake (f) as a result of the model acceleration. Momentumless wake (b) rapidly decays, while momentum wakes (c, f) remain active for a long time. The scale is given by the length of the model, which is seen in the first photograph. Time in the right column is measured from the beginning of acceleration. Visualization was performed using pH-indicator thymol blue (Ref. 6).

When a submerged self-propelled body moves with constant velocity (drag is balanced by thrust), at large distances the action of the body on fluid is equivalent to the action of a moving force doublet of intensity $Q (=Je$; J is the momentum source intensity, i.e. drag force or thrust, and e is the distance between the forces) and the wake has no net momentum. In a stratified fluid such a wake consists of a system of quadrupolar patches (Fig. 2, left) with zero net vorticity and zero net momentum. The resulting wake is not very energetic. When a self-propelled body accelerates or decelerates, the linear momentum $P (=Jt$; t is the duration of acceleration) is transmitted locally to the fluid. The action of the body on

the fluid in this case is equivalent to the action of a force doublet Q plus a force J . The wake has non-zero net momentum in the direction of the applied force. The disturbance produced by a force doublet decays much faster than the disturbance generated by a force (momentum source).

In order to clarify the basic physics of the large eddy formation, it is proposed to start with the simplest case of a single momentum source (jet) moving in a uniformly stratified flow. Experiments show that the formation of dipolar eddies occurs only in the stratified environment (compare Fig. 4, Fig. 4.1 and Fig. 5).

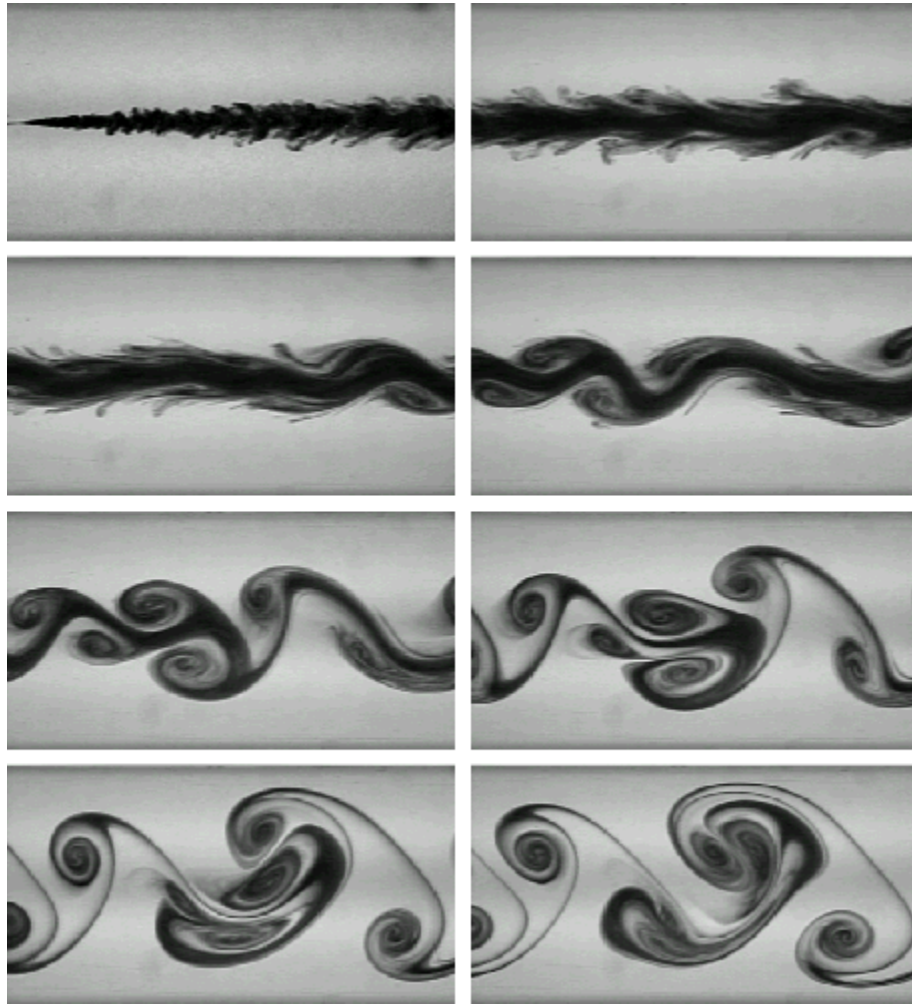


FIG. 4. Formation and doubling (through merging) of a vortex street in a stratified flow induced by a moving from right to left and continuously acting in the opposite direction momentum source (jet). Top view. The photographs were taken at $Nt = 0, 16, 32, 54.4, 78.4, 102.4, 134.4, 166.4$. Vertical size of each frame is 33 cm. Note that the direction of the resulting vortex street is opposite to that observed behind a towed body.

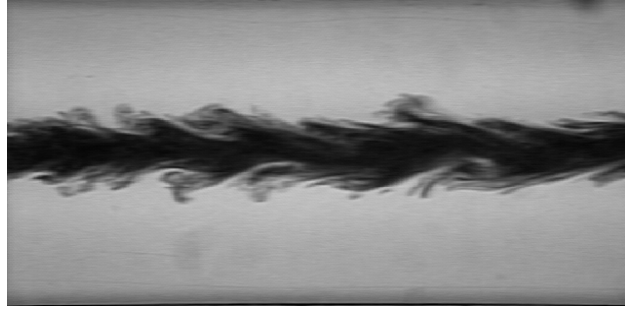


FIG. 4.1. Video sequence of images corresponding to Fig. 4. Time difference between adjacent frames $Nt = 16$.

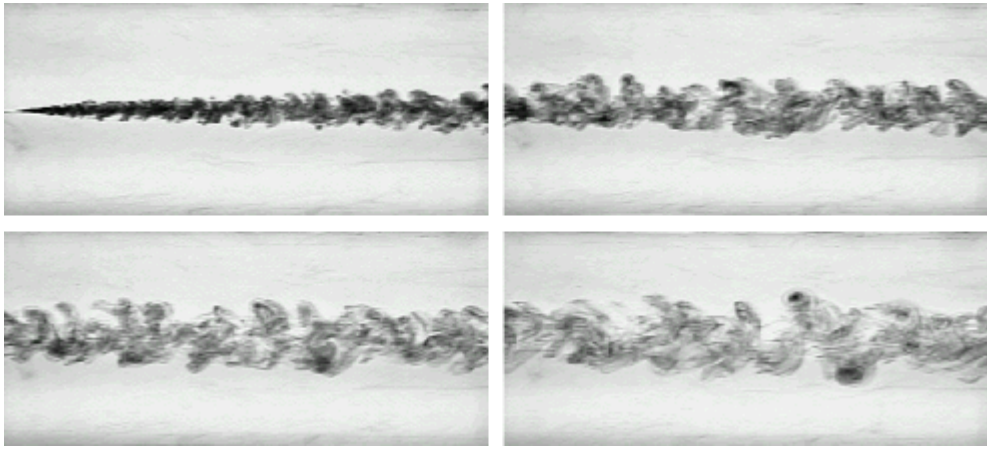


FIG. 5. Late flow development behind continuously acting momentum source (as in Fig. 4) moving from right to left in a homogeneous fluid ($N = 0$) and acting in the opposite direction. Top view. The photographs were taken at $t = 0, 10, 20, 34$ s. Vertical size of each frame is 33 cm. Note that the far wake consists of vortex rings which eventually dissipate due to the viscosity without the formation of a vortex street.

In general, the source can act either impulsively (during finite period of time) or continuously and move with variable velocity. The main interest here is to collect data on the mechanism and conditions of the eddy formation as well as their quantitative characteristics (formation time, lifetime, size) in terms of the critical system parameters which include dimensionless time Nt , Reynolds number $Re = J^{1/2}/\nu$, Froude number $Fr = U^2/NJ^{1/2}$ (J - momentum source intensity, U - translation velocity of the source, N - buoyancy frequency, and ν - fluid viscosity). It can be shown that the critical parameters thus defined directly correspond to the traditional Re_{body} and Fr_{body} numbers used for a towed body of the size D ¹¹. For example, for the problem of the localized moving momentum source one has

$$A_i = F_i(J, U, \nu, N, t), \quad (1)$$

where A_i is any flow characteristic and F_i is a function. Standard dimensional analysis shows that five variables can be reduced to three independent dimensionless parameters, which may be chosen as Re , Fr and Nt . At the same time, for the problem of towed solid body of the size D the momentum flux (drag force) produced by a towed body can be estimated as $J = (\rho/8) CU^2D^2$ (C is the drag coefficient). This gives: $Re_{\text{body}} = (8/C\rho)^{1/2}Re$ and $Fr_{\text{body}} = (C\rho/8)^{1/2}Fr$. Thus, the problem of a moving momentum source and the problem of a towed body are characterized by three similar dimensionless parameters. Experiments demonstrate that two different flow patterns may be realized in the late flow: (i) a system of large dipolar patches with finite momentum when the momentum source acts continuously (Fig. 4), or (ii) one unusually large dipole when momentum source acts impulsively (analog of accelerating self-propelled body, Fig. 3c).

It is possible to extend analytical results obtained for late momentum wakes¹ to the case of zero-momentum wakes and their combinations. Using Oseen's and modified boundary - layer approximations, one arrives to a linearized problem, which permits similarity solutions. Proper superposition of such solutions and standard limiting procedure allow one to calculate the velocity profiles $u(x,y,z)$ in late wakes generated by different forcing (force, force doublet, couple and their combinations). For example, for a force doublet of intensity Q moving horizontally with constant velocity U this gives

$$u(x,y,z) = -Q (4\pi n)^{-1} x^{-2} (h^2 - 1) \exp(-h^2), \quad (2)$$

(x and y are the along and across flow coordinates, $h = r (4n x/U)^{-1/2}$ is a self-similar variable). The solution, thus obtained, guarantees appropriate shape of the mean velocity profile (four inflection points in the considered case instead of two for a momentum wake) and, as a consequence, predetermines the number of shear layers inside the wake, which are responsible for the instability. The most important result here is that the amplitude of the velocity in (2) is specified by Q , that can be directly estimated in the experiment. Similar consideration can be done for more complicated forcing, and proper relations between the velocity amplitude and the forcing intensity can be found and verified experimentally. At present such general relations are absent in literature.

The results derived permit generalization, to include a non-zero lift force into consideration (Fig. 6).

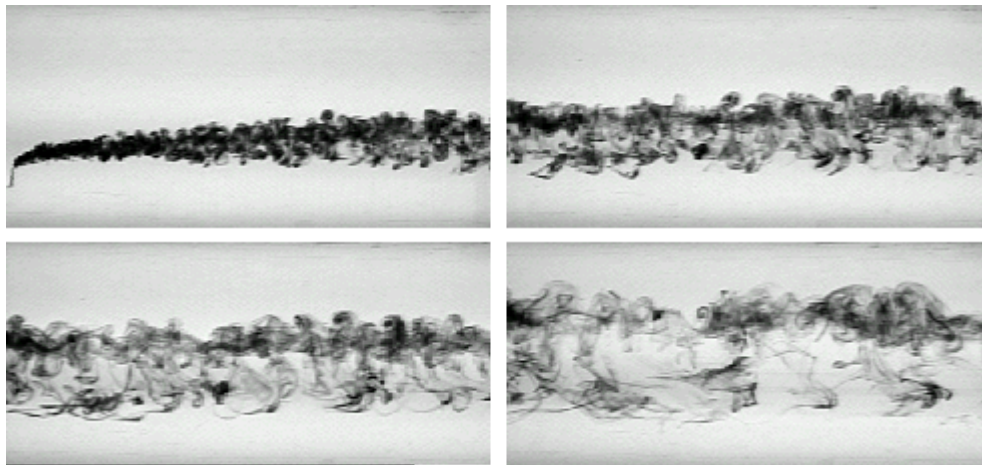


FIG. 6. Late flow development behind continuously acting momentum source (as in Fig. 4) moving from right to left in a homogeneous fluid ($N = 0$) and acting in the direction normal to the direction of its motion (lift force). Top view. The photographs were taken at $t = 0, 10, 20, 50$ s. Vertical size of each frame is 33 cm. The far wake is also three-dimensional, but has wider spatial dimension than in Fig. 5. There is no formation of a vortex street in the far flow field.

Research Objectives. The main objectives of this research are to answer the following scientific questions, which are deemed of the most importance for ocean engineering applications: (i) a delineation of source and environmental conditions under which large-scale stable eddies can be generated in the late-wake flows of maneuvering bodies, (ii) a determination of the formation time, lifetime, and other properties of these eddies as functions of the source and environmental parameters, and (iii) a characterization of these eddies based on their velocity, vorticity and density fields.

Ocean Engineering Relevance. Control, detection and monitoring of large eddies as characteristic signatures of the late-wake flow generated by steadily and unsteadily moving underwater vehicles.

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DIPOLAR EDDIES IN STRATIFIED SHEAR FLOW

Large coherent eddies (monopoles, dipoles and their combinations) are the persistent flow patterns, which are easily formed in stratified/rotating fluids. There is a particular interest in planar dipolar eddies because they possess a linear momentum and can propagate significant distances from the origin. Typical examples include large mushroom-like currents¹ and more complicated self-propagating eddies in the ocean². Another example is late stratified wakes behind towed/self-propelled bodies considered above.

In the ocean the horizontal shear typically is two to three orders of magnitude smaller than the vertical one. Therefore, the investigation of the effect of the background vertical shear on the dynamics of planar dipolar eddies has practical interest. Previous studies of this topic dealt only with the case of a co-planar (with the dipole plane) shear flow (horizontal³ or vertical⁴).

The experiments were conducted using 'Odell-Kovaszny' recirculating water channel to produce controllable vertical shear (see Ref.⁵). The tank has a glass test section 200x30x40 cm. Linearly stratified (by salt) water is driven around the channel by a viscous drag produced by two vertical stacks of thin plexiglas disks. The upper and lower parts of each stack rotate in opposite directions so that upper and bottom layers move horizontally in opposite directions too. The shear profiles of the horizontal velocity $u(z)$ (z is the vertical coordinate) were obtained by dropping sugar crystals in the test section. Vertical tracks from these crystals were visualized using a shadowgraph technique. The resulting shear $S = du/dz$ was almost constant in the central part of the test section, with approximately zero mean velocity at the mid-level. Vertical profiles of density (conductivity) were obtained using a standard four-electrode micro-scale conductivity probe. The central part of the resulting profiles was linear with approximately constant buoyancy frequency $N = [-g \rho_0^{-1} d\rho/dz]^{1/2}$ (g is the gravity acceleration, ρ_0 is the reference density).

Dipolar eddies with controllable intensity were generated by injecting during some time interval T_0 a small amount of dyed and neutrally buoyant fluid (volume flux q) from a thin horizontal nozzle (diameter $d = 0.15$ or 0.2 cm). The latter was located at the mid-level, where the mean velocity is zero. In this case the source of strong turbulent motion is equivalent to a concentrated momentum source of intensity J . The net momentum transported to the fluid during time interval T_0 is $I = JT_0$, where $J = q^2/s$ ($s = \rho d^2/4$) is the kinematic momentum flux. The images of the typical flow patterns were obtained by digitizing selected frames from the video.

Experiments demonstrate that the vertical shear may either significantly reduce (in comparison with the case of no shear) the lifetime of the formed eddies or completely suppress their formation (see Fig. 1 - 3).

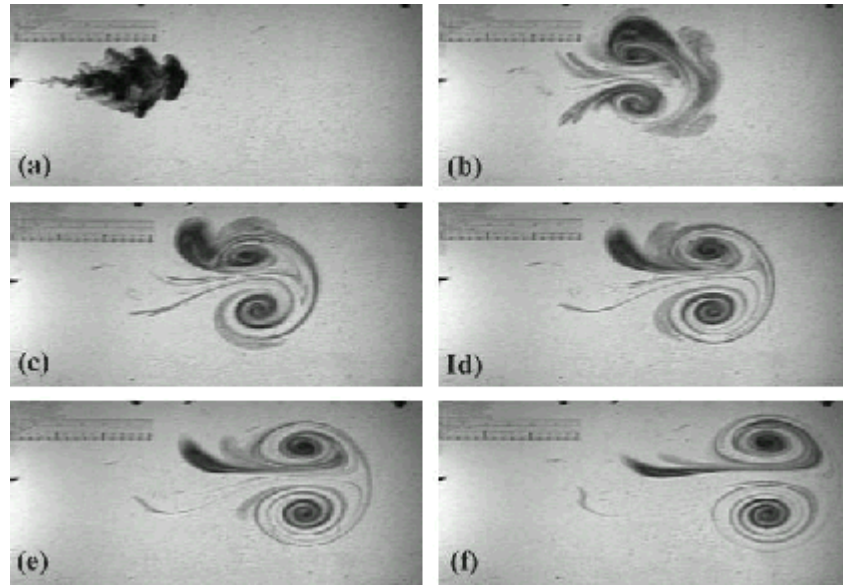


FIG. 1. Formation and evolution of dipolar eddy in a still stratified fluid. Top view. Horizontal jet acts impulsively along the tank axis. Parameters: $S = 0$, $N = 1.6 \text{ rad s}^{-1}$, $T_0 = 3.5 \text{ s}$, $Re = 440$. Time in seconds from the beginning of the experiment: 4, 27, 49, 71, 99, 161. The dipole always forms.

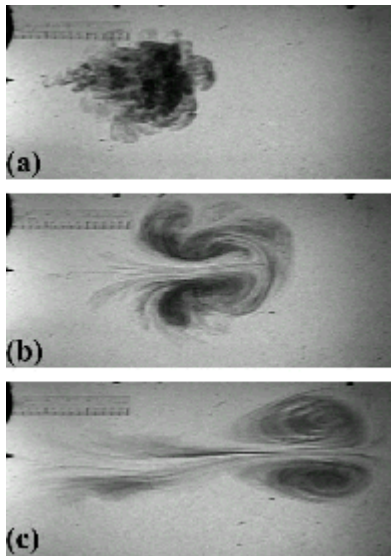


FIG. 2. Formation and evolution of dipolar eddy in a stratified fluid with shear. Top view. Horizontal jet acts impulsively along the tank axis. Parameters: $S = 0.25 \text{ s}^{-1}$, $N = 1.6 \text{ rad s}^{-1}$, $T_0 = 4.4 \text{ s}$, $Re = 950$. Time in seconds from the beginning of the experiment: 5, 19, 39, 79. The dipole is noticeably deformed by the shear.

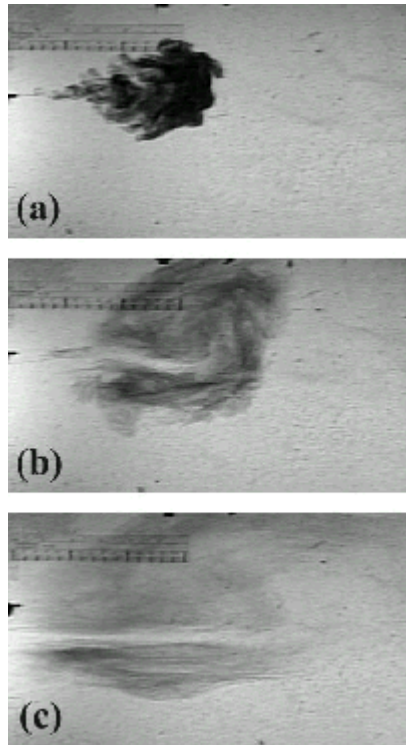


FIG. 3. Evolution of dipolar flow in a stratified fluid with strong shear. Top view. Horizontal jet acts impulsively along the tank axis. Parameters: $S = 0.5 \text{ s}^{-1}$, $N = 1.6 \text{ rad s}^{-1}$, $T_0 = 2.4 \text{ s}$, $Re = 1060$. Time in seconds from the beginning of the experiment: 3, 22, 53. The dipole doesn't form.

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